An Equilibrium Model of
Moving-average Predictability and Time-series Momentum

Guofu Zhou
Washington University in St. Louis

and

Yingzi Zhu
Tsinghua University[*]

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In an equilibrium model with rational informed investors and technical investors, we show that the moving average of past market prices can forecast the future price, explaining the strong predictive power found in many empirical studies. Our model can also explain the time series momentum that the market prices tend to be positively correlated in the short-run and negatively correlated in the long-run.

1 Introduction

Brock, Lakonishok, and LeBaron (1992) seems the first major academic study to provide convincing evidence on the stock market predictive power of moving averages of past prices, which are the key indicators of technical analysis that has been widely used by practitioners (e.g., Murphy, 1999, Schwager, 2012, and Lo and Hasanhodzic, 2010). Lo, Mamaysky, and Wang (2000) further strengthen the evidence with automated pattern recognition analysis. Recently, Neely, Rapach, Tu and Zhou (2013) find that technical indicators, primarily the moving averages, have forecasting power of the stock market matching or exceeding that of macroeconomic variables. Various rational models, such as Treynor and Ferguson (1985), Brown and Jennings (1989), Brock and Hommes (1998), Griffioen (2003), Chiarella, He and Hommes (2006), and Cespa and Vives (2012), show that past prices are useful for forecasting future prices under informational inefficiency. Behavioral models, such as DeLong, Shleifer, Summers, and Waldmann (1990) and Hong and Stein (1999), explain that behavior biases can lead to price trends, which can justify the value of moving averages that are trend-chasing tools. However, there are no equilibrium models that link the moving averages directly to future stock returns.

In this paper, we propose a continuous-time equilibrium model with hierarchical information structure. The model has two interesting implications. First, it provides a theoretical basis for using the moving averages in an explicit functional form, i.e., the average price divided by the current price. This functional form not only states that the moving average price is useful for forecasting the market return, but also it provides a measure of its impact. In contrast, the vast studies that use the moving averages rely on only an indicator function that indicates merely an up or down state of the market. The only exception is Han and Zhou (2013), who find the same specific functional form independently and intuitively. They show that sorting stocks according to the moving average functional can yield a new trend factor with an average return of about 1.61% per month more than twice the return on the well-known momentum factor (Jagadeesh and Titman, 1993), and a Sharpe ratio more than twice too.

The second implication of our model is that it offers an equilibrium explanation of the time series momentum (TSMOM) newly discovered by Moskowitz, Ooi and Pedersen (2012). They investigate a large number of asset classes, country equity indices, currencies, commodities and

bonds, and find that an asset's own past 12 month return predicts its future return in the next month. As pointed out by Moskowitz, Ooi and Pedersen (2012), while existing theories can offer explanations for the short-term positive and long-term negative return correlation pattern, they have two difficulties. First, the correlations of the TSMOM strategies across asset classes are larger than the correlations of the asset classes themselves. Second, very different types of investors in different asset markets are producing the same patterns at the same time. For example, in term of Hong and Stein (1999), if the under-reaction is due to slow information diffusion, then the speed for the information to spread out should be depending on the specific market mechanism, the distribution of investors, the coverage of analysts, trading and investment culture, etc. But the practically optimal lookback window for all the asset classes seems uniformly 12 months, indicating that the momentum phenomenon is kind of "technical" in the sense that it does not depend on the nature of specific form of information processing bias and dissemination mechanisms. Interestingly, this phenomenon fits exactly into our model where the technical investors learn from the markets by using moving averages of a lag length $L = 12$ months. When they trade with informed traders, their collective price impact will not be arbitraged away, and hence the equilibrium price will allow for a short-term positive return correlation within 12 months, and a long-term negative correlation beyond.

From a modelling perspective, our model follows closely Wang (1993) who provides the first dynamic asset pricing model under asymmetric information with a closed-form solution. As in Wang (1993), we assume that there are two types of investors, the informed ones and the uninformed ones. While the informed investors know more of the innovations of the economic fundamentals, the unformed ones only observe the dividends and prices. However, unlike Wang (1993), we assume that the unformed investors are the technical traders who utilize the historical prices via the moving average (MA) price instead of an optimal filter based on all the past prices. There are three reasons for this. First, with the optimal filter, it is not apparent theoretically the role of the moving average and other technical indicators, and the resulted model is unable to produce the earlier sign-reverting correlations. Second, many practitioners, who call themselves technical traders or non-discretionary traders, do use simple tools such as the moving averages to learn about the market and make their trading decisions accordingly. For example, a simple mechanical trading rule based on the moving averages can yield 70% correlation with the returns on managed futures industry (Burghardt,

Duncan and Liu, 2010), which is primarily trend-following and is one of the three best hedge fund styles that perform well in both good and bad times of the economy (Cao, Rapach and Zhou, 2013). Third, while learning via the MA is only suboptimal and bounded rational in the model, it is in fact a good robust strategy in general and can be better than a model-dependent optimal filter under model or parameter uncertainty (Zhu and Zhou, 2009). Given the technical investors who trade via the MA here, we, following Wang (1993) with some new techniques, also solve the equilibrium price along with other functions of interest in closed-form.

The intuition of our model is clear. When the technical investors choose to use the MA for investment decision making, the informed investors know about it and trade accordingly. Since the information updating rule used by technical investors is not the most efficient in the model, and given the uncertainty in asset supply for all investors, the informed investors cannot fully differentiate between the uncertainty in asset supply and technical investors' demand, and hence they cannot fully arbitrage away the impact of the technical trading on the price. As a result, the market equilibrium price can demonstrate a positive autocorrelation within the lag length of the MA rule. However, in the long-run, the price reverts to its mean level.

Conceptually, our model is closely related to DeLong, Shleifer, Summers, and Waldmann (1990). They also assume two types of investors: the rational speculator or "smart money" (referred to as George Soros) who knows the fundamentals and trade rationally as our informed traders, and the "positive feedback" traders whose trading decision is to follow price-trends (buy when the price is up and sell otherwise) as our technical traders. However, in their simple 4-period model, the role of the MA is unclear, neither the meaning of a trend. In contrast, the MA is an endogenous state variable in our model for the technical investors to make investment decisions and the MA is priced in equilibrium. As a result, its role in forecasting the market return is characterized explicitly as a simple functional of the MA level divided by the current price.

The paper proceeds as follows. In section 2 lays out the assumptions of the model. Section 3 presents the equilibrium solution. Section 4 analyzes the implications of the model. Section 5 examines how the model explains both the MA predictability and the TSMOM. Section 6 concludes.

2 The Model

Following Wang (1993), we consider an economy with a single risky asset which pays out its earnings as a random stream of dividend. There is a market for the asset which is traded as an infinitely divisible security and the market is populated with two types of traders: the well-informed rational investors and less informed technical investors who trade based on the MA signal (together with other public signals). As in Wang (1993), we assume a fluctuating total amount of security, which can be motivated by various realistic considerations, such as liquidity or noise trades, which serves mainly as a convenient modeling device to introduce uncertainty to the market so the equilibrium price is not fully revealing.

The information structure is hierarchical, that is, the information set of the informed investors encompasses that of the technical investors. The model is set in continuous-time with infinite horizon. Wang (1993) uses similar model to study the impact of information asymmetries on the time series of prices, risk premiums, price volatility and the negative autocorrelation in return. In contrast, our model, featuring the technical traders, aims at explaining the use of the MA and the short-run positive and long-run negative return correlations. Before pursuing the model any further, we address first two common motivation questions.

What are the theoretical reasons for the value of using the MA signal in the real world? The success of the MA depends on price trends which at least four types of models can explain. First, when investors do not receive information at the same time or heterogeneously informed, Treynor and Ferguson (1985) and Brown and Jennings (1989) demonstrate that past prices enable investors to make better inferences about future prices, and Grundy and McNichols (1989) and Blume, Easley, and O'Hara (1994) show that trading volume can provide useful information beyond prices. Second, if there is asset residual payoff uncertainty and/or persistence in liquidity trading, Cespa and Vives (2012) show that asset prices can deviate from their fundamental values and rational long-term investors follow trends. Third, due to behavioral biases, Hong and Stein (1999) explain that, at the start of a trend, investors underreact to news; as the market rises, investors subsequently overreact, leading to even higher prices. Fourth, with the presence of positive feedback traders (who buy after prices rise and sell after prices fall) as observed by hedge fund guru George Soros (2003), DeLong, Shleifer, Summers, and Waldmann (1990), and Edmans, Goldstein, and Jiang (2012) show that

there can be rational price trends. Empirically, Moskowitz, Ooi, and Pedersen (2012) find that pervasive price trends exist across commonly traded equity index, currency, commodity, and bond futures. Insofar as the stock market is not a pure random walk and exhibits periodic trends, the MA signal should prove useful the it is primarily designed to detect trends.[1]

Why do most investors in practice use the MA signal instead of using a time series model to capture price trends? The moving averages are simple and naive trend capturing tools. From behavior point of view, people at large prefer simple rules than complex ones. As put by the technical guru John Murphy (1999) in his famous book, "Moving average is one of the most versatile and widely used of all technical indicators. Because of the way it is constructed and the fact that it can be so easily quantified and tested, it is the basis for many mechanical trend-following systems in use today...... Chart analysis is largely subjective and difficult to test. As a result, chart analysis does not lend itself that well to computerization. Moving average rules, by contrast, can easily be programmed into a computer, which then generates specific buy and sell signals." However, the use of the MA seems quite rational too. A time series model such as an autoregressive process requires a large amount of stationary data to estimate accurately, but the real world data are non-stationary with changing regimes and parameters, and so it is often the case complex models underperform simple models out-of-sample. For examples, DeMiguel, Garlappi and Uppal (2009) show a simple equal-weighting portfolio rule beats sophisticated theory strategies, Timmermann (2006) and Rapach, Strauss and Zhou (2010) find that a simple average forecast provides better forecasts from complex econometric models. In term of the MA, Zhu and Zhou (2009) show that it is a robust approach compared with sophisticated ones.

Hence, the technical traders in our model are assumed to make their investment decisions using the MA. Similar assumptions are also made in Griffioen (2003), and Chiarella, He, and Hommes (2006). In our model, likely in practice too, the technical traders are synchronized and coordinated to form a critical mass, and their action do not cancel each other out.

Formally, we make below the same assumptions as Wang (1993), except Assumption 5 which specifies the MA as the learning tool for the technical traders instead of an optimal filter.

Assumption 1. The market is endowed with a certain amount of one risky asset, each unit of

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[1]Learning from market prices is valuable both theoretically and empirical not only to investors, but also to the government (see, e.g., Bernanke and Woodford, 1997, Bond and Goldstein, 2012, and references therein.)

which provides a dividend flow given by

$$dD_t = (\pi_t - \alpha_D D_t)dt + \sigma_D dB_{1t}, \tag{1}$$

where π_t is the mean level of dividend flow given by another stochastic process

$$d\pi_t = \alpha_\pi(\bar{\pi} - \pi_t)dt + \sigma_\pi dB_{2t}, \tag{2}$$

where we assume that B_{1t} and B_{2t} are independent.

This is a standard dividend process for the utility below to have closed-form solutions. When $\alpha_D > 0$, π_t/α_D is the short-run steady-state level of the dividends. The mean-reversion in π_t allows business cycles in the economy.

Assumption 2. The supply of the risky asset is $1 + \theta_t$ with

$$d\theta_t = -\alpha_\theta \theta_t dt + \sigma_\theta dB_{3t}, \tag{3}$$

where B_{3t} is another Brownian Motion independent from both B_{1t} and B_{2t}. Assumption 2 normalizes the long-run stationary level of the supply of the risky asset to 1, whereas θ_t represents shocks away from that level which implicitly allows liquidity trades outside the model.

Assumption 3. The claim on the risky asset is infinitely divisible and shares are held by the investors in the economy. Shares are traded in a competitive stock market with no transaction cost. The stock is the only security traded in the market. Let P be the equilibrium price of the stock.

Assumption 4. There is a risk-free investment to all investors with a constant rate of return $1 + r$ $(r > 0)$.

While the informed investors observe the mean grow rates of the dividends, the technical investors do not observe them. But both know the paths of dividends and price. However, the technical investors infer from the historical prices via the MA, that is,

$$A_t \equiv \int_{-\infty}^{t} \exp\left[-\alpha(t-s)\right]P_s ds, \tag{4}$$

with $\alpha > 0$. We use a exponentially weighted moving average rather than a simple moving average with sampling window to get close-form solution. The parameter α controls the size of moving average window. Note that

$$dA_t = (P_t - \alpha A_t)dt, \tag{5}$$

which is convenient for analytical analysis.

Assumption 5. There are two types of investors: the informed and the technical. The informed investors observe all state variables while technical investors only observe dividend and price. Formally, $\mathcal{F}^i(t) = \{D_\tau, P_\tau, \pi_\tau : \tau \leq t\}$ is the informed investors' information set at time t, and

$$(1, D_t, P_t, A_t)$$

is the technical investors' information set. Let w be the fraction of the uninformed investors.

Assumption 6. The structure of the market is common knowledge.

Assumption 7. The investors have expected additive utility with constant absolute risk aversion (CARA) conditional on their respective information set, $E[\int u(c(\tau), \tau)d\tau|\cdot]$, with

$$u(c(t), t) = -e^{-\rho t - c(t)}, \tag{6}$$

where ρ is the discount parameter and $c(t)$ is the consumption rate at time t.

3   Equilibrium

In this section, we solve for the equilibrium of the economy defined in previous section. The equilibrium concept is that of rational expectations developed by Lucas (1972), Green (1973), Grossman (1976), among others. Even though there is a certain bounded rationality due to cost to optimal learning, the model can still be viewed as a rational one because there is no resort to the use of any individual psychological bias.

The main result is the following:

Proposition: *In an economy defined in Assumptions 1-7, there exists a stationary rational expectations equilibrium. The equilibrium price function has the following linear form:*

$$P_t = p_0 + p_1 D_t + p_2 \pi_t + p_3 \theta_t + p_4 A_t, \tag{7}$$

where p_0, p_1, p_2, p_3 and p_4 are constants determined only by model parameters.

Interestingly, the equilibrium price, like Wang (1993), remains a linear function of the same state variables, except that A_t plays now the role of the estimation error of the dividend growth

π_t. Economically, it is clear that p_0, p_1 and p_2 are all positive, reflecting the positive price impact of the state variables. However, p_4 can be either positive or negative. Note that the solution to Equation (??) implies that

$$
\begin{aligned}
dA_t &= (P_t - \alpha A_t)dt \\
&= p_0 + p_1 D_t + p_2 \pi_t + p_3 \theta_t + (p_4 - \alpha)A_t.
\end{aligned}
\tag{8}
$$

Due to the stationarity of D_t, π_t and θ_t, it is easy to show that a solution exists when $p_4 < \alpha$. Our scheme of prove the proposition is to assume the price as given in (??), solve the optimal demand for the stock for both informed and technical investors, and by imposing the market clearing condition, we show the price exists in equilibrium by solving for parameters p_0, p_1, p_2, p_3 and p_4. We will provide the proof of the proposition next and then explore further its implications in Sections 4 and 5.

3.1   Informed Investor

We first describe the investment problem faced by informed investors given the price process in Equation (??). They face the investment opportunity defined by the excess return of one share of stock:

$$
dQ = (D - rP)dt + dP.
\tag{9}
$$

The information set and the investment opportunity for the informed investors are given in the following lemma:

Lemma 1: For informed investors the information set are given by the state variables $\Psi^i = (1, D_t, \pi_t, \theta_t, A_t)^T$, which satisfies the following SDE:

$$
d\Psi^i = e_\Psi^i \Psi^i dt + \sigma_\Psi^i dB_t^i,
\tag{10}
$$

where B_t^i is a 5-dimensional Brownian Motion, and $e_\Psi^i, \sigma_\Psi^i \in R^{5\times5}$ constant matrices, all of which are defined in Appendix ??. Further, the investment opportunity defined in Equation (??) for informed investors satisfies the stochastic differential equation:

$$
dQ = (D - rP)dt + dP = e_Q^i \Psi dt + \sigma_Q^i dB,
\tag{11}
$$

with $e_Q^i \in R^{5\times1}$ and $\sigma_Q^i \in R^{5\times1}$ which are given in Appendix ??.
QED.

The informed investors' optimization problem is

$$\max_{\eta^i, c^i} E\left[-\int_t^\infty e^{-\rho s - c(s)} ds | \mathcal{F}_t^i\right] \quad s.t. \quad dW^i = (rW^i - c^i)dt + \eta^i dQ. \tag{12}$$

Let $J^i(W^i, D_t, \pi_t, \theta_t, A_t; t)$ be the value function, then it satisfies the HJB equation

$$
\begin{aligned}
0 &= \max_{c, \eta} \left[-e^{-\rho t - c} + J_W(rW^i - c^i + \eta^i e_Q^i \Psi) + \frac{1}{2}\sigma_Q^i \sigma_Q^{iT} \eta^{i2} J_{WW} + \eta^i \sigma_Q^i \sigma_\Psi^{iT} J_{W\Psi}\right] \\
&\quad -\rho J + (e_\Psi^i \Psi)^T J_\Psi + \frac{1}{2}\sigma_\Psi^i J_{\Psi\Psi} \sigma_\Psi^{iT}.
\end{aligned} \tag{13}
$$

The solution to the optimization problem is provided in the following theorem:

Theorem 1: Equation (**??**) has a solution of the form:

$$J^i(W^i, D_t, \pi_t, \theta_t, A_t; t) = -e^{-\rho t - rW - \frac{1}{2}\Psi^{iT} V^i \Psi^i}, \tag{14}$$

with $\Psi^i = (1, D_t, \pi_t, \theta_t, A_t)^T$, and $V^i \in R^{5\times 5}$ a positive definite symmetric matrix. The optimal demand for stock is given by

$$\eta^i = f^i \Psi^i = f_0^i + f_1^i D_t + f_2^i \pi_t + f_3^i \theta_t + f_4^i A_t, \tag{15}$$

where $f_0^i, f_1^i, f_2^i, f_3^i$ and f_4^i are constants.

Proof. See Appendix **??**.

The theorem says that, given the model assumptions, the informed investors' demand for stock is a simple linear function of the fundamental variables, D_t, π_t, θ_t, as well as the technical variable A_t. The result is very similar to the one in Wang (1993) except that the estimation error there is now replaced by A_t, the trading signal of the technical traders.

3.2 Technical Investors

Technical investors are in a different situation from the informed ones in that they face different information set and different perceived investment opportunity. In particular, they do not observe state variable π_t as the informed investors do. However, they do know the dynamics of the processes. In our setting, they use MA signal of past stock prices together with the other observables (D_t, P_t) to infer π_t, that is, they learn about π_t from its projection onto their information set $\Psi^u = (1, D_t, P_t, A_t)$. Specifically, they use the following linear regression to infer π, denoted as π^u,

$$\pi_t^u = \beta_0 + \beta_1 D_t + \beta_2 P_t + \beta_3 A_t + \sigma_u u_t. \tag{16}$$

9

With the insights from Wang (1993), we assume that the technical investors, who don't observe π_t, know the process that drives its dynamics, and hence they can infer the unconditional linear regression coefficients in Equation (??). The technical details for computing the parameters $\beta_0, \beta_1, \beta_2, \beta_3$ and σ_u are given in the Appendix ??.

Given the price in Equation (??), the technical investors' estimation of the state variable θ_t, defined as θ_t^u, can be inferred from the price via

$$
\begin{aligned}
\theta_t^u &= \frac{1}{p_3}[P_t - (p_0 + p_1 D_t + p_2 \pi_t^u + p_4 A_t)] \\
&= \gamma_0 + \gamma_1 D_t + \gamma_2 P_t + \gamma_3 A_t - \frac{p_2}{p_3}\sigma_u u_t,
\end{aligned}
\tag{17}
$$

where the parameters $\gamma_0, \gamma_1, \gamma_2$ and γ_3 are given in Appendix ??.

Define

$$
\begin{aligned}
\hat{\pi}_t &= \beta_0 + \beta_1 D_t + \beta_2 P_t + \beta_3 A_t, \tag{18}\\
\hat{\theta}_t &= \gamma_0 + \gamma_1 D_t + \gamma_2 P_t + \gamma_3 A_t. \tag{19}
\end{aligned}
$$

The dynamics of D_t for technical investor is then

$$
\begin{aligned}
dD_t &= (\hat{\pi}_t + \sigma_u u_t - \alpha_D D_t)dt + \sigma_D dB_{1t} \\
&= (\hat{\pi}_t - \alpha_D D_t)dt + \sigma_D dB_{1t} + \sigma_u dZ_t \\
&= (\hat{\pi}_t - \alpha_D D_t)dt + \hat{\sigma}_D dB_{1t}^u,
\end{aligned}
\tag{20}
$$

where Z_t is defined as $Z_t = \int_0^t u_s ds$, which is another independent Brownian motion with u_t the white noise in regression equation (??). The third equality in Equation (??) has used

$$
\hat{\sigma}_D dB_{1t}^u = \sigma_D dB_{1t} + \sigma_u dZ_t,
\tag{21}
$$

with

$$
\hat{\sigma}_D^2 = \sigma_D^2 + \sigma_u^2.
\tag{22}
$$

We define another state variable

$$
\Lambda_t = p_2 \pi_t + p_3 \theta_t,
\tag{23}
$$

which is observable by technical investors through observing the equilibrium price and dividend.

We have

$$
\begin{aligned}
d\Lambda_t &= (p_2\alpha_\pi(\bar\pi - \hat\pi_t) - p_3\alpha_\theta\hat\theta_t)dt \\
&\quad + p_2(\sigma_\pi dB_2 - \alpha_\pi\sigma_u dZ_t) + p_3(\sigma_\theta dB_3 + \alpha_\theta\frac{p_2}{p_3}\sigma_u dZ_t) \\
&= (p_2\alpha_\pi(\bar\pi - \hat\pi_t) - p_3\alpha_\theta\hat\theta_t)dt + \hat\sigma_\Lambda dB_{2t}^u,
\end{aligned}
\tag{24}
$$

with

$$
\begin{aligned}
\hat\sigma_\Lambda dB_{2t}^u &= p_2(\sigma_\pi dB_{2t} - \alpha_\pi\sigma_u dZ_t) + p_3(\sigma_\theta dB_{3t} + \alpha_\theta\frac{p_2}{p_3}\sigma_u dZ_t) \\
&= p_2\sigma_\pi dB_{2t} + p_3\sigma_\theta dB_{3t} + (\alpha_\theta - \alpha_\pi)p_2\sigma_u dZ_t
\end{aligned}
\tag{25}
$$

and

$$
\hat\sigma_\Lambda^2 = (p_2\sigma_\pi)^2 + (p_3\sigma_\theta)^2 + (\alpha_\theta - \alpha_\pi)^2 p_2^2\sigma_u^2.
\tag{26}
$$

Based on Equations (**??**) and (**??**), the correlation between dB_{1t}^u and dB_{2t}^u, defined as

$$
\mathrm{Var}(dB_{1t}^u, dB_{2t}^u) \equiv \varrho dt,
$$

can be written as

$$
\varrho = \frac{p_2\sigma_u^2(\alpha_\theta - \alpha_\pi)}{\hat\sigma_D\hat\sigma_\Lambda}.
\tag{27}
$$

With the above discussion, we summarize the investment environment faced by the technical traders as in the following lemma:

Lemma 2: The state variable set observed by technical investors, $\Psi^u = (1, D_t, P_t, A_t)^T$, follows a stochastic differential equation

$$
d\Psi^u = e_\psi^u\Psi^u dt + \sigma_\Psi^u dB_t^u,
\tag{28}
$$

where $B_t^u = (0, B_{1t}^u, B_{2t}^u, 0)$, with B_{1t}^u and B_{2t}^u defined in Equations (**??**) and (**??**), and e_Ψ^u and σ_Ψ^u given by

$$
e_\Psi^u = \begin{pmatrix}
0 & 0 & 0 & 0 \\
\beta_0 & \beta_1 - \alpha_D & \beta_2 & \beta_3 \\
q_0 & q_1 & q_2 & q_3 \\
0 & 0 & 1 & -\alpha
\end{pmatrix}, \quad
\sigma_\Psi^u = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & \hat\sigma_D & 0 & 0 \\
0 & p_1\hat\sigma_D + \varrho\hat\sigma_\Lambda & \sqrt{1 - \varrho^2}\hat\sigma_\Lambda & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}.
\tag{29}
$$

The investment opportunity is

$$
\begin{aligned}
dQ &= (D - rP)dt + dP \\
&= e_Q^u\Psi^u dt + \sigma_Q^u dB_t^u,
\end{aligned}
$$

11

with e_Q^u and σ_Q^u defined as

$$e_Q^u = \begin{pmatrix} q_0 & 1+q_1 & q_2-r & q_3 \end{pmatrix}, \quad \sigma_Q^u = \begin{pmatrix} 0 & p_1\hat{\sigma}_D + \varrho\hat{\sigma}_\Lambda & \sqrt{1-\varrho^2}\hat{\sigma}_\Lambda & 0 \end{pmatrix}. \tag{30}$$

Proof: See Appendix **??**.

Note that, technically, in order to derive a linear equilibrium price in closed form, the demand function, and hence the investment opportunity set, should be in the state variable set. By adding a state variable A_t to the functional form of the price, Equation (**??**) guarantees that the investment opportunity set is in the state variable set. To solve the technical investors' optimization problem, let W^u be the wealth of a technical investor's wealth, η^u be the holding of stock and c^u be the consumption. Then the investor's optimization problem is

$$\max_{\eta^u, c^u} E\left[-\int_t^\infty e^{-\rho s - c(s)} ds | \mathcal{F}_t^u\right] \quad s.t. \quad dW = (rW^u - c^u)dt + \eta^u dQ. \tag{31}$$

Let $J^u(W^u, \Psi^u; t)$ be the value function, then it solves the following HJB equation,

$$\begin{aligned}
0 &= \max_{c^u, \eta^u} \left[-e^{-\rho t - c^u} + J_{W^u}^u(rW^u - c^u + \eta^u e_Q^u \Psi^u) + \frac{1}{2}\sigma_Q^u \sigma_Q^{uT} \eta^{u2} J_{W^u W^u}^u + \eta^u \sigma_Q^u \sigma_\Psi^{uT} J_{W^u \Psi^u}^u\right] \\
&\quad - \rho J^u + (e_\Psi^u \Psi^u)^T J_\Psi^u + \frac{1}{2}\sigma_\Psi^u J_{\Psi^u \Psi^u}^u \sigma_\Psi^{uT}. \tag{32}
\end{aligned}$$

The solution is provided by

Theorem 2: Equation (**??**) has a solution of the following form,

$$J^u(W^u, D_t, P_t, A_t; t) = -e^{-\rho t - rW^u - \frac{1}{2}\Psi^{uT}V^u\Psi^u}, \tag{33}$$

with

$$\Psi^u = (1, D_t, P_t, A_t)^T, \tag{34}$$

and $V^u \in R^{4\times 4}$ a positive definite symmetric matrix. The optimal demand for stock is given by

$$\eta^u = f_0^u + f_1^u D_t + f_2^u P_t + f_3^i A_t, \tag{35}$$

where f_0^u, f_1^u, f_2^u and f_3^i are constants.

Proof. See Appendix **??**.

Theorem 2 says that the technical investors' demand for stock is a linear function of state variables, D_t, P_t, observable to them, and A_t, the technical indicator they use. Note that, in

contrast, the price P_t is not in the demand function of the informed investors because, to them, θ_t and P_t are equivalent in terms of information content. On the other hand, technical investors observe neither π_t nor θ_t, and hence they can only pin down their demand through the price function P_t. Indeed, to them, P_t provides new information.

3.3 Market Clearing

Given Equations (??) and (??), the demands of stock by informed and technical investors, the market clearing condition requires

$$\eta^i + \eta^u = 1 + \theta_t,$$

or equivalently,

$$(1 - w)[f_0^i + f_1^i D_t + f_2^i \pi_t + f_3^i \theta_t + f_4^i A_t] + w[f_0^u + f_1^u D_t + f_2^u P_t + f_3^u A_t] = 1 + \theta_t. \tag{36}$$

Substitute P_t in (??) into above, we obtain

$$(1 - w)f_0^i + w(f_0^u + f_2^u p_0) = 1,$$
$$(1 - w)f_1^i + w(f_1^u + f_2^u p_1) = 0,$$
$$(1 - w)f_2^i + w(f_2^u p_2) = 0, \tag{37}$$
$$(1 - w)f_3^i + w(f_2^u p_3) = 1,$$
$$(1 - w)f_4^i + w(f_3^u + f_2^u p_4) = 0.$$

The solution to Equation (??) determines the coefficients p_0, p_1, p_2, p_3 and p_4 for the price function of (??). It is easy to show that a unique solution exists under general conditions. This completes the proof of the main proposition of our paper.

If all the investors are informed, i.e., $w = 0$, there is an explicit solution to the problem with the parameters in Equation (??) given as

$$p_0 = \Phi + p_0^* = \frac{\alpha_\pi \bar{\pi}}{r(r + \alpha_D)(r + \alpha_\pi)} - \left[\frac{\sigma_D^2}{(r + \alpha_D)^2} + \frac{\sigma_\pi^2}{(r + \alpha_D)^2(r + \alpha_\pi)^2} \right],$$
$$p_1 = p_D^* = \frac{1}{r + \alpha_D}, \tag{38}$$
$$p_2 = p_\pi^* = \frac{1}{(r + \alpha_D)(r + \alpha_\pi)},$$

$p_3 < 0$ and $p_4 = 0$. This reduces to the solutions previously given by Campbell and Kyle (1993) and Wang (1993).

Other than the special case, there are no explicit solutions to the constants in general. But the associated Ricatti algebraic equations can be easily solved by using the Matlab function CARE, and then the solution to the equations (??) can be solved via any nonlinear least square solver.

4   Price, Trading and Comparative Statics

In this section, we examine the properties and implications of the equilibrium price. To better characterize the solution, we re-arrange Equation (??) as

$$P_t = p + p_D D_t + p_\pi \pi_t + p_\theta \theta_t + p_{mv}(P_t - \alpha A_t), \tag{39}$$

with

$$p_0 = \frac{p}{1 - p_{mv}}, \ \ p_1 = \frac{p_D}{1 - p_{mv}}, \ \ p_3 = \frac{p_\pi}{1 - p_{mv}}, \ \ p_4 = -\frac{\alpha p_{mv}}{1 - p_{mv}}. \tag{40}$$

Since we are looking for the stationary solution for P_t, the last term of Equation (??) has mean 0. Notice that the last term in Equation (??) is a well-known technical indicator called MACD, the moving average convergence/divergence indicator created by Gerald Appel in the late 1970s, which is used to spot changes in the strength, direction, momentum, and duration of a trend in a stock's price. Intuitively, given

$$dA_t = (P_t - \alpha A_t)dt,$$

the MACD can be understood as the "speed" of the MA: when it is positive (negative), the trend of the MA is moving up (down). If there is indeed a trend or predictability in the price return (as in this model), the MACD is indeed effective to detect the trend by filtering out the noise term. Note that p_{mv} needs to be less than 1 to make the solution stationary. When $p_{mv} > 0$, the price process in (??) indicates that the MA rule exhibits a trend-following type of behavior; while when $p_{mv} < 0$, the MA signal is used for reverse trading.

In the following subsections, we characterize the equilibrium price in our model, examining the impact of information structure, the trading strategy and the comparative statics of the model parameters.

4.1 Stock Price and Information Structure

In our model, the parameter w characterizes the information structure of the economy. By changing w, we can see how information structure can impact on the prices. The base case parameter is set at

$$r = 0.05, \rho = 0.2, \bar{\pi} = 0.85, \sigma_D = 1.0, \sigma_\pi = 0.6, \sigma_\theta = 3.0, \alpha_\pi = 0.2, \alpha_\theta = 0.4, \alpha_D = 1.0.$$

Table **??** presents the numerical solution, examining price impact of information structure. When $w = 0$, the numerical solver gives the same the parameters as the explicit formula given in Equation (**??**). Panel A of Table **??** shows the (re-arranged) equilibrium price parameters in Equation (**??**) with the base case parameters for $\alpha = 1$. The price sensitivity to D_t, measured by p_D, increases from 0.9526 to 2.3489 when the ratio of technical investors increases from 0 to 1, while the price sensitivity to π_t, measured by p_π, decreases from 3.8095 to 0. In indeed, the price does not depend on π_t when no one in the market can observe it. The price sensitivity to θ_t's, the parameter p_θ, is negative and increasing in magnitude as the number of technical investors increases. This is due to the fact that technical investors infer the combination of $\Lambda \equiv p_\pi \pi + p_\theta \theta$ from the price. If they assign smaller portion of Λ to π, they must infer from the price that a larger portion of the Λ is contributed by θ, hence larger (in magnitude) the p_θ. The constant parameter p is not monotone in relation to w, it increases from 6.8209 at $w = 0$ to 21.6172 at $w = 0.5$, and then decreases to 4.8052 when $w = 1$.

The most interesting part is the price sensitivity to MACD, the parameter of p_{mv}, which is relatively small due to the low regression parameter β_3 in the regression of (**??**) that the technical investors use to infer π_t, making the technical investors more conservative in asset allocation based on MA. A caveat is that, the small sensitivity to MA in the regression, and hence small p_{mv}, which is due to the assumption of zero correlation among state variables, is not an important feature of the model. We can otherwise assume that the state variables are correlated, and/or that the technical traders use the MA rule in a more prominent way, but these various assumptions do not alter the main insights of the model that MA signal is priced. Moreover, if we increase the MA window and set $\alpha = 0.1$, the price sensitivity to MACD is much increased, as demonstrated in Panel B of the table.

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4.2 Trading Strategy

To understand the price behavior, specifically the term involving MACD, we need to understand the trading strategy of each group of investors. In our setting, we define trend-following (contrarian) strategy as a positive (negative) feedback strategy in the sense of DeLong, Shleifer, Summers, and Waldmann (1990). Hence, a strategy positively (negatively) correlated to the MACD, or equivalently negatively (positively) correlated to A_t, is a trend-following (contrarian) strategy. It is important to keep in mind that the equilibrium price solution is a stationary one, hence the aggregate behavior of the market has to be contrarian, otherwise the price will blow up (evidenced by, eg, the market crash of 1987 when trend-following trade due to portfolio insurance increased dramatically). When market is dominated by informed investors, the market price can sustain certain portion of trend-following strategy by technical investors as long as the majority, the informed investors, are contrarian; when market is dominated by technical investor, they have to be contrarian otherwise the price blows up in the long run.

We examine a numerical example in Table (**??**). It is important to notice the sign of p_{mv}. When $\alpha = 1$, p_{mv} is positive for small w and becomes negative when w increases to 0.3. The positive p_{mv} implies a trend-following price behavior, while negative p_{mv} implies a contrarian. The portfolio demand by informed and technical investors are presented in Panel A of Table **??**. In the table, we define the portfolio demand in terms of

$$MACD_t = P_t - \alpha A_t,$$

and

$$\Lambda_t = P_t - (p - p_D D_t - p_{mv} MACD_t),$$

hence the portfolio demand by informed investors of Equation (**??**) and technical investors of Equation (**??**) can be expressed as

$$\eta^i = g_0^i + g_1^i D_t + g_2^i \pi_t + g_3^i \theta_t + g_4^i MV_t,$$
$$\eta^u = g_0^u + g_1^u D_t + g_2^u \Lambda_t + g_3^u MV_t,$$

where g^i's and g^u's are the demand loading on state variables. It demonstrates clearly that the positive (negative) p_{mv} in Table **??** ($\alpha = 1$) is driven by positive (negative) demand by technical investors' portfolio loading on the MACD. To examine what is driving the sign of g_3^u, we note

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that g_3^u is approximately equal to $q_3 \approx p_1\beta_3$, where $p_1 > 0$, hence it has the same sign as β_3, the regression slope of π_t on A_t in Equations (??). Panel B of the table shows numerically the regression coefficients of Equation (??) and (??) used by technical investors. Indeed, it shows that when w is relatively small, β_3 is negative, which says that high (low) A_t (relative to P_t) implies low (high) value π_t, hence the demand from technical investor for the MACD is negative, and the technical investors portfolio demand exhibit trend-following behavior; on the other hand, when w is bigger, high (low) A_t implies high (low) π_t, the technical investors exhibit contrarian behavior. Note that the informed investors are always on the opposite side of the trade to the technical, exhibiting the opposite trading behavior.

In summary, the intuition from the model is that, when the weight of the technical investors are small, the usual trend following MA strategy is profitable; when more investors are using the trend following MA strategy, it becomes unprofitable.

4.3 Comparative Statics

In this subsection, we examine the how the model parameters impact on the equilibrium price. Table ?? examines the comparative statics of the price with respect to the persistence parameter for π, α_π. In this table, we set $\alpha = 0.1$. Panel A shows the case for $\alpha_\pi = 0.1$ while Panel B the base case with $\alpha_\pi = 0.2$. This parameter has big impact on price. For example, when $w = 0.5$, $p = -5.6116$ for $\alpha_\pi = 0.1$, and $p = 16.2126$ for $\alpha_\pi = 0.2$. When π_t is more persistent with smaller α_π, the price is more sensitive to π_t and θ_t, while less sensitive to MACD.

Table ?? examines the impact of parameter σ_π on the price. Panel A is based on $\sigma_\pi = 0.8$ and Panel B the base case parameter of $\sigma_\pi = 0.6$. There are two points to notice. First, the price is much reduced for increased σ_π due to higher risk premium. Second, the market can sustain more technical investors with trend following trading strategy. p_{mv} is positive up to $w = 0.7$ in Panel A. This is due to the fact that the technical investors are more conservative in using MACD signal when π_t is more volatile and the regression R^2 is smaller.

Table ?? examines the impact of MACD coefficient α on price. We set $w = 0.1$ with all base case parameters except for $\sigma_\pi = 0.8$. The main impact of α is on the sensitivity to MACD signal. p_{mv} decreases dramatically from 0.0234 to 0.0007 when α increases from 0.1 to 12.

Table **??** examines the implication of w and α on the stock price volatility and the risk premium. From (**??**), the price process

$$dP_t = p_1 dD_t + p_2 d\pi_t + p_3 d\theta_t + p_4 dA_t = \mu_p dt + \sigma_p dZ_t, \tag{41}$$

with the instantaneous volatility defined as

$$\sigma_p = \sqrt{(p_1\sigma_D)^2 + (p_2\sigma_\pi)^2 + (p_3\sigma_\theta)^2}. \tag{42}$$

Recall the investment opportunity is

$$dQ = (D - rP)dt + dP = e_Q\Psi dt + \sigma_Q dB_t. \tag{43}$$

Following Wang (1993), we can define the risk premium as $e_Q\Psi/P$. Since both the numerator and denominator are time-varying, we take the average

$$\bar{P} = p + p_D\bar{\pi} + p_\pi\bar{\pi}, \tag{44}$$

$$\bar{\Psi} = (1, \bar{\pi}, \bar{\pi}, 0, \bar{P}/\alpha)^T. \tag{45}$$

Then the risk premium is

$$RP = \frac{e_Q\bar{\Psi}}{\bar{P}}, \tag{46}$$

where e_Q is defined in Equation (**??**). Table **??** presents the σ_p and RP in terms of varying w and α. The price volatility increases as w increases, while the RP is not monotone as it varies in the same way as p.

5 MA Predictability and Momentum

In this section, we show first that the MA divided by price is theoretically a predictor of the stock returns, and then explain why the lagged stock returns can also predict the returns, a phenomenon of the times series momentum.

5.1 Predictability of MA

The main implication of our model is that the stock price can be predicted by MA signal. Specifically, taking finite difference in Equation (**??**) with discrete time interval Δt, and using Equation (**??**), we obtain

$$\Delta P_t = p_1 \Delta D_t + p_2 \Delta \pi_t + p_3 \Delta \theta_t + p_4(P_t - \alpha A_t)\Delta t.$$

After dividing by price, we have the following predictive regression on the moving average,

$$r_{t+1} = \alpha + \beta \frac{A_t}{P_t} + \epsilon_t, \tag{47}$$

where impact of other variables are summarized in the noise term of the regression. In empirical applications, A_t are easily approximated by the simple moving average

$$A_t = \frac{1}{L} \sum_{i=0}^{L-1} P_{t-i\Delta t}, \tag{48}$$

where L is lag length or the moving average window.

Theoretically, our model states that A_t/P_t should be a predictor of the market return. If the model is applied to asset classes, similar conclusions hold. However, in the real world, the proportion of technical traders can change over time, and hence the slope on A_t/P_t may not always be positive.

Table **??** reports the predictive regression of monthly returns on the S&P500 stock index on the daily moving average prices of the index over January 1963 and December 2012. The starting month of the regression is the first month after January 1963 for which A_t is computable with lag $L = 10$ days and up to 200 days, which are the popular lag lengths used in practice and in Brock, Lakonishok, and LeBaron (1992). It is interesting that the slopes are all positive, indicating that trend-following is the right strategy in practice for the technical traders. When $L = 10$ and $L = 100$, the results are statistically significant at the 1% and 5% levels, respectively. This may suggest that there are both short-term and long-term trend-followers, and not many in between. The statistical evidence is remarkable since the stock market as a whole is known notoriously very difficult to predict. Very often, the evidence of any predictor is much stronger by applying to cross-section stocks or portfolios and then aggregating together.

Indeed, cross-sectionally, Han and Zhou (2013) run similar regressions by using all of the moving averages, and find that the resulted trend factor is both highly statistically and economically significant. It not only more than doubles the average return of the well-known momentum factor (Jagadeesh and Titman, 1993) which occurs in global markets too (Hou, Karolyi and Kho, 2011), but also explains much better the cross-section returns. As reviewed in the introduction, there are various economic reasons for why the market or stocks can trend. Our paper not only provides a new theoretical explanation based on the presence of technical investors, but also shows explicitly how the moving average can be used to forecast future returns.

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5.2 Time Series Momentum

The time series momentum (Moskowitz, Ooi and Pedersen, 2012) that the past 12 month return predicts that of the next month can be understood easily if we can show that the model allows for short-term positive price correlation at a lag of L about one year and long-term negative correlation beyond. Mathematically, it suffices to show that the price autocovariance has such a pattern.

Analytically, the autocovariance of the price in our is

$$< P_{t+\tau} - P_t, P_t - P_{t-\tau} >$$
$$= A_D g(\alpha_D, \tau) + B_D g(\alpha_1, \tau) + A_\pi g(\alpha_\pi, \tau) + B_\pi g(\alpha_1, \tau) + A_\theta g(\alpha_\theta, \tau) + B_\theta g(\alpha_1, \tau), \quad (49)$$

where the parameters $A_D, B_D, A_\pi, B_\pi, A_\theta, B_\theta$ are derived in the Appendix **??**, and

$$g(\alpha, \tau) = -(1 - e^{-\alpha \tau})^2.$$

To see why fully rational model such as Wang (1993) can only generate negative autocorrelation[2], note that in Wang (1993), the parameters of B_D, B_π and B_θ are all zeroes because those are interaction terms between state variables and the MA signal, and the signs of A_D, A_π and A_θ are the same. Hence, the autocorrelations always have the same sign for any τ in Wang (1993). In our model, all parameter A's and B's can have different signs due to the interaction between state variable mean reversion speed and the lag length of the MA signal. In particular, the parameter p_4 needs to be positive to generate this autocovariance pattern. We show numerically an example in Figure **??**. The autocovariance is positive up till $\tau = 0.5$ year and then become negative for longer horizons. This is largely consistent with the momentum literature that past winners over 6 to 12 month outperforms over the next 3–12 months.

Panel A of Table **??** provides the regression results for returns on S&P500 on the lagged returns over January 1963 and December 2012. While the slopes on past 2 and 6 month returns are negative, the slope on the lagged 12 month return is positive that is consistent with Moskowitz, Ooi and Pedersen's (2012) TSMOM finding (the results are similar if their slightly different time periods of the data are used). Although the slope is not statistically significant, Moskowitz, Ooi and Pedersen (2012) show that a portfolio of more asset classes, which are of the same pattern, will

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[2]Wang (1993) can generate either positive or negative autocovariance for one parameter set, but cannot generate the pattern of short-run positive and and long-run negative autocovariance for any set of parameter.

be both economically and statistically significant. In light of our model, a possible explanation for the strong continuation of the TSMOM is that there are substantial number of technical traders across asset classes who use moving averages of the same lag length of roughly one year.

More generally, one may imagine that there are momentum traders with different investment horizons in the real world, so that their time series predictive lags are different. Then we may run the same regression on past returns of many lag lengths. To avoid over fitting the regression, we can impose a simple restriction that the slopes are the same on all lags up to L. This amounts to regressing the returns on the moving average of past returns with the lag length L. Panel B of Table ?? reports the results. The statistical significance is much greater than before. This interesting result may suggest a new way to improve the time series momentum even further.

6 Conclusions

This paper provides a theoretical explanation to both predictability of the moving average (MA) of past stock prices and the time series momentum (TSMOM). While the MA predictability has been studied by researchers and utilized by practitioners for a long time, the TSMOM was only discovered recently by Moskowitz, Ooi and Pedersen (2012). Empirically, both of them provide today perhaps the strongest evidence on predictability and abnormal returns. Our equilibrium explanation is based on an economy in which there are both rational informed investors and technical investors who use MA as one of their trading signals. In our model, the price is not fully revealing, and so the MA signal is priced risk factor which depends on the population ratio of technical traders in the market. When the population ratio is small, technical investors will behave as trend-followers. However, when the population ratio is large, they can be contrarians. The model not only explains the predictability of the MA, but also identifies explicitly the functional form of the MA that predicts the market. In addition, the model can also explain Moskowitz, Ooi and Pedersen (2012)'s TSMOM that the market prices tend to be positively correlated in the short-run and negatively correlated in the long-run, and that the past 12 month return predicts the future next month return across asset classes.

Since our exploratory model has a number of simplifying assumptions, it will be of interest to extend the model by relaxing some of them. For examples, it will be important to allow the portion

of technical traders to change over time. In practice, it is almost always the case that a new trend, a bubble in particular, starts with relatively few investors, then it attracts more and more trend-followers over time, and eventually the trend-followers vanish completely as the trend reverses. It will also be important to allow different types of technical traders, arbitragers, market-makers, and investors of various horizons, as it is the case in the real world.

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A   Appendix

In this Appendix, we provide detailed proofs of the results.

A.1   Proof of Lemma 1

The state variables of the economy is given by

$$dD_t = (\pi_t - \alpha_D D_t)dt + \sigma_D dB_{1t},$$

$$d\pi_t = \alpha_\pi(\bar{\pi} - \pi_t)dt + \sigma_\pi dB_{2t}, \tag{A1}$$

$$d\theta_t = -\alpha_\theta \theta_t dt + \sigma_\theta dB_{3t}.$$

The set that determines the informed investors' opportunity set is

$$\Psi = (1, D_t, \pi_t, \theta_t, A_t)^T, \tag{A2}$$

which satisfies the following vector SDE,

$$d\Psi = e_\Psi \Psi dt + \sigma_\Psi dB_t, \tag{A3}$$

where B_t is a 5-dimensional Brownian Motion, e_Ψ and $\sigma_\Psi \in R^{5\times 5}$ are constant matrices,

$$e_\Psi = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha_D & 1 & 0 & 0 \\ \alpha_\pi \bar{\pi} & 0 & -\alpha_\pi & 0 & 0 \\ 0 & 0 & 0 & -\alpha_\theta & 0 \\ p_0 & p_1 & p_2 & p_3 & p_4 - \alpha \end{pmatrix}, \tag{A4}$$

and

$$\sigma_\Psi = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_D & 0 & 0 & 0 \\ 0 & 0 & \sigma_\pi & 0 & 0 \\ 0 & 0 & 0 & \sigma_\theta & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A5}$$

The investment opportunity is then

$$dQ = (D_t - rP_t)dt + dP_t \equiv e_Q \Psi dt + \sigma_Q dB_t, \tag{A6}$$

by differentiating Equation (**??**), where e_Q and σ_Q are

$$e_Q = (p_0(p_4 - r) + p_2\alpha_\pi\bar{\pi}, 1 + p_1(p_4 - r - \alpha_D), p_1 + p_2(p4 - r - \alpha_\pi), p_3(p_4 - r - \alpha_\theta), p_4(p_4 - r - \alpha)),$$
$$\tag{A7}$$

and

$$\sigma_Q = \begin{pmatrix} 0 & p_1\sigma_D & p_2\sigma_\pi & p_3\sigma_\theta & 0 \end{pmatrix}. \tag{A8}$$

The result implies Lemma 1. QED.

A.2   Proof of Theorem 1

To prove Theorem 1, we conjecture a solution for the portfolio demand of the informed investors as a linear function of state variables as in Equation (??), and conjecture accordingly the value function be of the following form,

$$J^i(W^i, D_t, \pi_t, \theta_t, A_t; t) = -e^{-\rho t - rW - \frac{1}{2}\Psi^{iT} V^i \Psi^i}. \tag{A9}$$

Substituting this into the HJB equation, we obtain

$$\eta = f^i \Psi, \tag{A10}$$

where

$$f^i = \frac{1}{r}(\sigma_Q \sigma_Q^T)^{-1}(e_Q - \sigma_Q \sigma_\Psi^T V^i) \tag{A11}$$

with V^i a symmetric positive satisfying

$$V^i \sigma_\Psi \sigma_\Psi^T V^{iT} - (\sigma_Q \sigma_Q^T)^{-1}(e_Q - \sigma_Q \sigma_\Psi^T V^i)^T(e_Q - \sigma_Q \sigma_\Psi^T V^i) + rV^i - (e_\Psi^T V^i + V^i e_\Psi) + 2k\delta_{11}^{(5)} = 0 \tag{A12}$$

, $k \equiv [(r - \rho) - r \ln r] - \frac{1}{2}Tr(\sigma_\Psi^T \sigma_\Psi V^i)$ and

$$[\delta_{(11)}^{(5)}]_{ij} = \begin{cases} 1, & i = j = 1 \\ 0, & \text{otherwise.} \end{cases} \tag{A13}$$

This yields Theorem 1. QED.

A.3   Computation of β_i's and γ_i's

Based on (??), the regression slope, $\beta \equiv (\beta_1, \beta_2, \beta_3)$, is given by

$$\beta = Var^{-1} \cdot Cov, \tag{A14}$$

where Var $\in R^{3\times3}$ and $\mu \in R^{3\times1}$ be the variance and mean of vector $Y_t = (D_t, P_t, A_t)$, and Cov $\in R^{1\times3}$ the covariance between π_t and (D_t, P_t, A_t). Consider first how how compute $Var(Y_t)$

and $Cov(\pi_t, Y_t)$. Given the price relation (??), we have $Y = FX$ with

$$F \equiv \begin{pmatrix} 0 & 0 & 1 & 0 \\ p_2 & p_3 & p_1 & p_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{A15}$$

Then it follows that

$$\text{Var} = FCF^T, \quad \mu = Fm^T, \quad \text{Cov} = e_1 CF^T, \tag{A16}$$

where $e_1 \equiv (1, 0, 0, 0)$.

To compute the mean and covariance matrix of $X_t = (\pi_t, \theta_t, D_t, A_t)$, we, based on (??) and (??), have the following dynamics,

$$
\begin{aligned}
d\pi_t &= \alpha_\pi(\bar{\pi} - \pi_t)dt + \sigma_\pi dB_{2t}, \\
d\theta_t &= -\alpha_\theta \theta_t dt + \sigma_\theta dB_{3t}, \\
dD_t &= (\pi_t - \alpha_D D_t)dt + \sigma_D dB_{1t}, \\
dA_t &= [p_0 + p_1 D_t + p_2 \pi_t + p_3 \theta_t + (p_4 - \alpha)A_t]dt,
\end{aligned}
$$

which is an affine system. Let $\alpha_1 \equiv \alpha - p_4$. We now consider the following transform of X_t,

$$\Phi(u, x, t, T) = E_t[e^{u \cdot X_T}] = e^{A(t) + B(t) \cdot X_t}, \tag{A17}$$

where $A(t)$ and $B(t)$ satisfy the ODE system (Duffie, Pan and Singleton 2000) below,

$$\frac{dB(t)}{dt} = -K_1^T B(t), \quad B(T) = u, \tag{A18}$$

$$\frac{dA(t)}{dt} = -K_0 \cdot B(t) - \frac{1}{2} B(t)^T H_0 B(t), \quad A(T) = 0, \tag{A19}$$

with

$$K_0 = \begin{pmatrix} \alpha_\pi \bar{\pi} \\ 0 \\ 0 \\ p_0 \end{pmatrix}, \quad K_1 = \begin{pmatrix} -\alpha_\pi & 0 & 0 & 0 \\ 0 & -\alpha_\theta & 0 & 0 \\ 1 & 0 & -\alpha_D & 0 \\ p_2 & p_3 & p_1 & -\alpha_1 \end{pmatrix}, \quad H_0 = \begin{pmatrix} \sigma_\pi^2 & 0 & 0 & 0 \\ 0 & \sigma_\theta^2 & 0 & 0 \\ 0 & 0 & \sigma_D^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A20}$$

Note that the elements of covariance matrix of X_t are the quadratic terms of X_t in Taylor expansion of Equation (??).

To solve (??) and (??) analytically, we denote by U and Λ the eigenvectors and eigenvalues of K_1^T, ie,

$$UK_1^T = \Lambda U, \quad U^{-1}U = UU^{-1} = I, \quad \Lambda = diag(\lambda_1, ..., \lambda_4). \tag{A21}$$

Then, due to the special form of K_1^T, the eigenvalues are all negative,

$$\lambda_1 = -\alpha_\pi, \quad \lambda_2 = -\alpha_\theta, \quad \lambda_3 = -\alpha_D, \quad \lambda_4 = -\alpha_1.$$

Therefore, we have

$$B_t = U^{-1} e^{\Lambda(T-t)} U u, \tag{A22}$$

and

$$
\begin{aligned}
A_t &= \int_0^t K_0^T B_s ds + \frac{1}{2} \int_0^t B_s^T H_0 B_s ds \\
&= K_0^T U^{-1} \left[\int_0^t e^{\Lambda s} ds \right] U u + \frac{1}{2} (U u)^T \left[\int_0^t e^{\Lambda s} (U^{-1})^T H_0 U^{-1} e^{\Lambda s} ds \right] U u, \tag{A23}
\end{aligned}
$$

where we have used (??).

Since we are only interested in the limit case when $T \to \infty$, and the elements of Λ are negative, the only term with non-zero limit in the exponent of Equation (??) is the second term of A_t in Equation (??). To compute the second term of A_t, we define

$$H \equiv \int_0^t e^{\Lambda s} (U^{-1})^T H_0 U^{-1} e^{\Lambda s} ds.$$

It is easy to show that the elements of H, denoted as H_{ij} for $i, j = 1, 2, 3, 4$, can be computed as

$$H_{ij} = -\frac{1}{\lambda_i + \lambda_j} [(U^{-1})^T H_0 U^{-1}]_{ij}. \tag{A24}$$

Then the covariance matrix C of X_t and mean of X_t can be written as

$$C = U^T H U, \quad m = -K_0^T U^{-1} \Lambda^{-1} U.$$

In addition, the mean and covariance of Y_t can be computed as in Equation (??). Then the regression coefficients in (??) can be readily computed. Moreover,

$$\beta_0 = m e_1^T - \beta^T \mu, \qquad \sigma_u^2 = e_1 C e_1^T - \beta^T (\text{Var}) \beta. \tag{A25}$$

Once the coefficients β_i's are determined, by matching the coefficients of both sides of

$$p_2 \hat{\pi}_t + p_3 \hat{\theta}_t = p_2 \pi_t + p_3 \theta_t = P_t - p_0 - p_1 D_t - p_4 A_t, \tag{A26}$$

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we obtain

$$p_2\beta_0 + p_3\gamma_0 = p_0,$$

$$p_2\beta_1 + p_3\gamma_1 = -p_1,$$

$$p_2\beta_2 + p_3\gamma_2 = 1,$$

$$p_2\beta_3 + p_3\gamma_3 = -p_4.$$

(A27)

Hence, we find

$$\gamma_0 = \frac{-p_0 - p_2\beta_0}{p_3}, \quad \gamma_1 = \frac{-p_1 - p_2\beta_1}{p_3}, \quad \gamma_2 = \frac{1 - p_2\beta_2}{p_3}, \quad \gamma_3 = \frac{-p_4 - p_2\beta_3}{p_3}. \tag{A28}$$

This accomplishes the task.

A.4 Proof of Lemma 2

Given $\hat{\pi}_t$ and $\hat{\theta}_t$ in (**??**) and (**??**), and the dynamics of Λ_t in Equation (**??**), we obtain

$$
\begin{aligned}
dP_t &= p_1 dD_t + d\Lambda_t + p_4 dA_t \\
&= p_1(\hat{\pi}_t - \alpha_D D_t)dt + p_1\hat{\sigma}_D dB_{1t}^u + d\Lambda_t + p_4 dA_t \\
&= [q_0 + q_1 D_t + q_2 P_t + q_3 A_t]dt + p_1\hat{\sigma}_D dB_{1t}^u + \hat{\sigma}_\Lambda dB_{2t}^u,
\end{aligned}
\tag{A29}
$$

where

$$
\begin{aligned}
q_0 &= p_1\beta_0 + p_2\alpha_\pi(\bar{\pi} - \beta_0) - \alpha_\theta p_3\gamma_0, \\
q_1 &= p_1(\beta_1 - \alpha_D) - p_2\alpha_\pi\beta_1 - p_3\alpha_\theta\gamma_1, \\
q_2 &= p_1\beta_2 - p_2\alpha_\pi\beta_2 - p_3\alpha_\theta\gamma_2 + p_4, \\
q_3 &= p_1\beta_3 - p_2\alpha_\pi\beta_3 - p_3\alpha_\theta\gamma_3 - p_4\alpha.
\end{aligned}
$$

Applying further Equations (**??**), we obtain

$$
\begin{aligned}
q_0 &= p_1\beta_0 + p_2\alpha_\pi\bar{\pi} + p_0(\alpha_\pi + \alpha_\theta), \\
q_1 &= p_1(\beta_1 - \alpha_D + \alpha_\pi + \alpha_\theta), \\
q_2 &= p_1\beta_2 + (p_4 - \alpha_\pi - \alpha_\theta), \\
q_3 &= p_1\beta_3 + p_4(\alpha_\pi + \alpha_\theta - \alpha).
\end{aligned}
\tag{A30}
$$

The parameters $\hat{\sigma}_D^2$, $\hat{\sigma}_\Lambda^2$ and ϱ are defined in Equations (**??**), (**??**) and (**??**), and σ_u^2 is defined in Equation (**??**).

Combined with Equation (**??**) and

$$dA_t = (P_t - \alpha A_t)dt, \tag{A31}$$

we obtain the dynamics for $\Psi^u = (1, D_t, P_t, A_t)^T$, which follows the following SDE,

$$d\Psi^u = e_\psi^u \Psi^u dt + \sigma_\Psi^u dB_t^u, \tag{A32}$$

where

$$e_\psi^u = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \beta_0 & \beta_1 - \alpha_D & \beta_2 & \beta_3 \\ q_0 & q_1 & q_2 & q_3 \\ 0 & 0 & 1 & -\alpha \end{pmatrix}, \tag{A33}$$

and

$$\sigma_\psi^u = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \hat{\sigma}_D & 0 & 0 \\ 0 & p_1\hat{\sigma}_D + \varrho\hat{\sigma}_\Lambda & \sqrt{1 - \varrho^2}\hat{\sigma}_\Lambda & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A34}$$

The investment opportunity is

$$dQ = (D_t - rP_t)dt + dP_t \equiv e_Q^u \Psi dt + \sigma_Q^u dB_t, \tag{A35}$$

obtained via Equation (**??**), where e_Q^u and σ_Q^u are given by

$$e_Q^u = \begin{pmatrix} q_0 & 1+q_1 & q_2 - r & q_3 \end{pmatrix}, \tag{A36}$$

and

$$\sigma_Q^u = \begin{pmatrix} 0 & p_1\hat{\sigma}_D + \varrho\hat{\sigma}_\Lambda & \sqrt{1 - \varrho^2}\hat{\sigma}_\Lambda & 0 \end{pmatrix}. \tag{A37}$$

Then we have the lemma. QED.

A.5   Proof of Theorem 2

To prove Theorem 2, we conjecture a solution for the portfolio demand by the informed investors as linear function of state variable set Ψ as in Equation (**??**), and conjecture accordingly the value function

$$J^u(W^u, D_t, P_t, A_t; t) = -e^{-\rho t - rW - \frac{1}{2}\Psi^{uT}V^u\Psi^u}. \tag{A38}$$

Substituting this into the HJB equation, we obtain

$$\eta = f^u \Psi^u, \tag{A39}$$

where

$$f^u = \frac{1}{r}(\sigma_Q \sigma_Q^T)^{-1}(e_Q - \sigma_Q \sigma_\Psi^T V^u) \tag{A40}$$

with V^u a symmetric positive definite matrix satisfying

$$V^u \sigma_\Psi \sigma_\Psi^T V^{uT} - (\sigma_Q \sigma_Q^T)^{-1}(e_Q - \sigma_Q \sigma_\Psi^T V^u)^T(e_Q - \sigma_Q \sigma_\Psi^T V^u) + rV^u - (e_\Psi^T V^u + V^u e_\Psi) + 2k\delta_{11}^{(4)} = 0, \tag{A41}$$

$k \equiv [(r - \rho) - r \ln r] - \frac{1}{2}Tr(\sigma_\Psi^T \sigma_\Psi V^u)$ and

$$[\delta_{(11)}^{(4)}]_{ij} = \begin{cases} 1, & i = j = 1 \\ 0, & \text{otherwise.} \end{cases} \tag{A42}$$

This implies Theorem 2. QED.

A.6 Computation for the Autocovariance

We want to compute the autocovariance of the zero cost stock return defined as

$$< P_{t+\tau} - P_t, P_t - P_{t-\tau} > .$$

It suffices to compute the autocovariance of $< P_{t+\tau}, P_t >$ for any $\tau > 0$. Note that the solution to the MA signal A_t, according to Equation (??), is

$$A_t = \int_{-\infty}^t e^{(p_4 - \alpha)(t-s)}(p_0 + p_1 D_s + p_2 \pi_s + p_3 \theta_s)ds, \tag{A43}$$

that is, A_t is the moving average of the state variables D_t, π_t and θ_t which are stationary. Note that in our model, we do not assume a priori any fixed correlation among the state variables, hence the autocovariance of can be compute as

$$< P_{t+\tau}, P_t >=< F_{t+\tau}^D, F_t^D > + < F_{t+\tau}^\pi, F_t^\pi > + < F_{t+\tau}^\theta, F_t^\theta >, \tag{A44}$$

where

$$\begin{aligned} F_t^D &\equiv D_t + p_4 \int_{-\infty}^t D_s e^{-\alpha_1(t-s)}ds, \\ F_t^\pi &\equiv \pi_t + p_4 \int_{-\infty}^t \pi_s e^{-\alpha_1(t-s)}ds, \\ F_t^\theta &\equiv \theta_t + p_4 \int_{-\infty}^t \theta_s e^{-\alpha_1(t-s)}ds, \end{aligned}$$

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and $\alpha_1 = \alpha - p_4$. Hence, we have

$$
\begin{aligned}
&< F^D_{t+\tau}, F^D_t > \\
&= k_D \left\{ e^{-\alpha_D \tau} + p_4 \left[\frac{e^{-\alpha_D \tau}}{\alpha_1 + \alpha_D} + \frac{e^{-\alpha_1 \tau}}{\alpha_1 + \alpha_D} + \frac{e^{-\alpha_1 \tau} - e^{-\alpha_D \tau}}{\alpha_D - \alpha_1} \right] + p_4^2 \left[\frac{e^{-\alpha_1 \tau}}{(\alpha_D + \alpha_1)\alpha_1} + \frac{(e^{-\alpha_1 \tau} - e^{-\alpha_D \tau})}{(\alpha_1 + \alpha_D)(\alpha_D - \alpha_1)} \right] \right\},
\end{aligned}
$$

where $k_D = \sigma_D^2 / 2\alpha_D$. The formula for $< F^\pi_{t+\tau}, F^\pi_t >$ and $< F^\theta_{t+\tau}, F^\theta_t >$ are similar with α_D (σ_D) replaced by α_π (σ_π) and α_θ (σ_θ), respectively. Now, defining

$$
g(\alpha, \tau) = -(1 - e^{-\alpha \tau})^2,
$$

we obtain the autocovariance as

$$
\begin{aligned}
&< P_{t+\tau} - P_t, P_t - P_{t-\tau} > \\
&= p_1^2 [A_D g(\alpha_D, \tau) + B_D g(\alpha_1, \tau)] \\
&\quad + p_2^2 [A_\pi g(\alpha_\pi, \tau) + B_\pi g(\alpha_1, \tau)] \\
&\quad + p_3^2 [A_\theta g(\alpha_\theta, \tau) + B_\theta g(\alpha_1, \tau)],
\end{aligned} \tag{A45}
$$

where

$$
\begin{aligned}
A_D &= k_D \left[1 - \frac{p_4(2\alpha_1 + p_4)}{(\alpha_1 + \alpha_D)(\alpha_D - \alpha_1)} \right], \\
B_D &= k_D \frac{p_4(2\alpha_1 + p_4)}{(\alpha_1 + \alpha_D)(\alpha_D - \alpha_1)} \frac{\alpha_D}{\alpha_1},
\end{aligned}
$$

and A_π, A_θ, B_π, and B_θ are similarly defined.

Table 1: Stock Price vs w

The table shows the impact of the fraction of technical investors on the equilibrium stock price,

$$P_t = p + p_D D_t + p_\pi \pi_t + p_\theta \theta_t + p_{mv}(P_t - \alpha A_t).$$

The parameters are $r = 0.05, \rho = 0.2, \bar\pi = 0.85, \sigma_D = 1.0, \sigma_\pi = 0.6, \sigma_\theta = 3.0, \alpha_\pi = 0.2, \alpha_\theta = 0.4, \alpha_D = 1.0$. The two panels present the results for two different moving average windows measured by $1/\alpha$ with $\alpha = 1$ and 0.1, respectively.

w	p	p_D	p_π	p_θ	p_{mv}
A. The case of $\alpha = 1$					
0	6.8209	0.9524	3.8095	-1.0803	0.0000
0.1	8.9806	1.0148	3.6265	-1.1449	0.0003
0.2	12.0439	1.0887	3.4124	-1.2129	0.0002
0.3	16.0629	1.1758	3.1629	-1.2841	-0.0004
0.4	20.1075	1.2777	2.8731	-1.3586	-0.0016
0.5	21.6172	1.3971	2.5385	-1.4362	-0.0038
0.6	18.9471	1.5362	2.1533	-1.5158	-0.0070
0.7	14.2068	1.6978	1.7122	-1.5968	-0.0118
0.8	9.9461	1.8845	1.2094	-1.6763	-0.0182
0.9	6.8820	2.1002	0.6403	-1.7516	-0.0268
1	4.8052	2.3489	0.0000	-1.8180	-0.0382
B. The case of $\alpha = 0.1$					
0	6.8209	0.9524	3.8095	-1.0803	0.0000
0.1	8.6479	1.0225	3.6589	-1.1644	-0.0070
0.2	10.9427	1.1069	3.4747	-1.2565	-0.0152
0.3	13.4936	1.2093	3.2539	-1.3589	-0.0256
0.4	15.6019	1.3338	2.9918	-1.4736	-0.0395
0.5	16.2126	1.4825	2.6763	-1.5996	-0.0560
0.6	14.9448	1.6605	2.3011	-1.7382	-0.0764
0.7	12.5443	1.8719	1.8561	-1.8883	-0.1013
0.8	10.0112	2.1212	1.3313	-2.0469	-0.1312
0.9	7.8738	2.4191	0.7181	-2.2140	-0.1696
1	6.2033	2.7752	0.0000	-2.3826	-0.2180

Table 2: Asset Allocation by Informed vs Technical Traders

The table shows the allocations of the informed and technical investors,

$$\eta^i = g_0^i + g_1^i D_t + g_2^i \pi_t + g_3^i \theta_t + g_4^i MV_t,$$
$$\eta^u = g_0^u + g_1^u D_t + g_2^u \Lambda_t + g_3^u MV_t,$$

where $MV_t = P_t - \alpha A_t$, and $\Lambda_t = P_t - (p - p_D D_t - p_{mv} MV_t)$. The parameters are $r = 0.05, \rho = 0.2, \bar{\pi} = 0.85, \sigma_D = 1.0, \sigma_\pi = 0.6, \sigma_\theta = 3.0, \alpha_\pi = 0.2, \alpha_\theta = 0.4, \alpha_D = 1.0$, and $\alpha = 1$. Panel B shows the updating rule of Equation (**??**) and (**??**) used by technical investors.

A. Allocation Coefficients (g)

w	Informed					Technical			
	1	D_t	π_t	θ_t	MV	1	D_t	Λ_t	MV
0	1.0000	0.0000	0.0000	1.0000	0.0000	4.2075	1.0010	-0.4780	0.0007
0.1	0.6012	-0.1062	0.1957	1.0493	-0.0005	4.5892	0.9561	-0.4855	0.0044
0.2	0.0278	-0.2253	0.4184	1.1013	-0.0004	4.8887	0.9011	-0.4903	0.0014
0.3	-0.7446	-0.3583	0.6713	1.1560	0.0006	5.0706	0.8359	-0.4955	-0.0014
0.4	-1.5700	-0.5066	0.9582	1.2136	0.0026	4.8554	0.7599	-0.5010	-0.0039
0.5	-1.9856	-0.6719	1.2832	1.2740	0.0058	3.9855	0.6719	-0.5074	-0.0057
0.6	-1.6424	-0.8560	1.6516	1.3373	0.0104	2.7621	0.5707	-0.5149	-0.0070
0.7	-0.8740	-1.0618	2.0697	1.4032	0.0168	1.8028	0.4550	-0.5242	-0.0072
0.8	-0.1508	-1.2931	2.5460	1.4714	0.0251	1.2877	0.3233	-0.5358	-0.0063
0.9	0.3729	-1.5559	3.0921	1.5410	0.0359	1.0697	0.1729	-0.5510	-0.0040
1	0.7192	-1.8601	3.7258	1.6109	0.0498	1.0000	0.0000	-0.5711	0.0000

B. Updating Rule by Technical Traders

w	γ_0	γ_1	γ_2	γ_3	β_0	β_1	β_2	β_3
0	7.1220	2.0024	-0.6175	-0.0010	0.2292	0.3178	0.0874	-0.0003
0.1	8.6724	1.9929	-0.6187	-0.0064	0.2615	0.3493	0.0804	-0.0019
0.2	10.6822	1.9652	-0.6247	-0.0021	0.2672	0.3794	0.0710	-0.0007
0.3	13.1496	1.9249	-0.6283	0.0022	0.2602	0.4098	0.0612	0.0008
0.4	15.2974	1.8712	-0.6293	0.0065	0.2352	0.4401	0.0511	0.0025
0.5	15.4069	1.8036	-0.6271	0.0105	0.2009	0.4700	0.0406	0.0044
0.6	12.7588	1.7223	-0.6217	0.0140	0.1826	0.4991	0.0300	0.0066
0.7	9.1019	1.6282	-0.6129	0.0169	0.1907	0.5269	0.0194	0.0089
0.8	6.0865	1.5232	-0.6011	0.0191	0.2122	0.5529	0.0088	0.0114
0.9	4.0147	1.4099	-0.5868	0.0204	0.2345	0.5768	-0.0017	0.0141
1	2.6432	1.2921	-0.5711	0.0210	0.2524	0.5986	-0.0121	0.0169

Table 3: Stock Price vs α_π

The table shows the impact of the persistence of long run dividend level state variable α_π on the equilibrium stock price,

$$P_t = p + p_D D_t + p_\pi \pi_t + p_\theta \theta_t + p_{mv}(P_t - \alpha A_t).$$

The parameters are $r = 0.05, \rho = 0.2, \bar{\pi} = 0.85, \sigma_D = 1.0, \sigma_\pi = 0.6, \sigma_\theta = 3.0, \alpha_\theta = 0.4, \alpha_D = 1.0, \alpha = 0.1$. The two panels present the results for two different α_π for various w, the fraction of technical investors.

w	p	p_D	p_π	p_θ	p_{mv}
A. The case of $\alpha_\pi = 0.1$					
0	-4.6257	0.9524	6.3491	-7.6464	0.0000
0.1	-4.8919	1.1443	5.9320	-8.1713	-0.0050
0.2	-5.1220	1.3519	5.4653	-8.6938	-0.0091
0.3	-5.3205	1.5766	4.9541	-9.2211	-0.0132
0.4	-5.4817	1.8171	4.3926	-9.7421	-0.0163
0.5	-5.6116	2.0754	3.7850	-10.2645	-0.0194
0.6	-5.7112	2.3522	3.1298	-10.7872	-0.0225
0.7	-5.7757	2.6456	2.4227	-11.2981	-0.0246
0.8	-5.8118	2.9579	1.6664	-11.8057	-0.0267
0.9	-5.8207	3.2900	0.8594	-12.3092	-0.0288
1	-5.7971	3.6386	0.0000	-12.7928	-0.0299
B. The case of $\alpha_\pi = 0.2$					
0	6.8209	0.9524	3.8095	-1.0803	0.0000
0.1	8.6479	1.0225	3.6589	-1.1644	-0.0070
0.2	10.9427	1.1069	3.4747	-1.2565	-0.0152
0.3	13.4936	1.2093	3.2539	-1.3589	-0.0256
0.4	15.6019	1.3338	2.9918	-1.4736	-0.0395
0.5	16.2126	1.4825	2.6763	-1.5996	-0.0560
0.6	14.9448	1.6605	2.3011	-1.7382	-0.0764
0.7	12.5443	1.8719	1.8561	-1.8883	-0.1013
0.8	10.0112	2.1212	1.3313	-2.0469	-0.1312
0.9	7.8738	2.4191	0.7181	-2.2140	-0.1696
1	6.2033	2.7752	0.0000	-2.3826	-0.2180

Table 4: Stock Price vs σ_π

The table shows the impact of moving average parameter α on the equilibrium stock price,

$$P_t = p + p_D D_t + p_\pi \pi_t + p_\theta \theta_t + p_{mv}(P_t - \alpha A_t).$$

The parameters are $r = 0.05, \rho = 0.2, \bar{\pi} = 0.85, \sigma_D = 1.0, \alpha_\pi = 0.2, \sigma_\theta = 3.0, \alpha_\theta = 0.4, \alpha_D = 1.0$, and $\alpha = 0.1$. The moving average window is measured by $1/\alpha$. The two panels present the results for two different σ_π's for various w, the fraction of technical investors.

			A. The case of $\sigma_\pi = 0.8$		
w	p	p_D	p_π	p_θ	p_{mv}
0	2.7574	0.9524	3.8095	-3.0589	0.0000
0.1	2.7623	1.0569	3.5380	-3.2505	0.0157
0.2	2.6579	1.1810	3.2649	-3.4967	0.0253
0.3	2.4217	1.3270	2.9796	-3.8034	0.0301
0.4	2.0582	1.4982	2.6754	-4.1803	0.0301
0.5	1.5911	1.6969	2.3432	-4.6341	0.0263
0.6	1.0538	1.9250	1.9751	-5.1708	0.0196
0.7	0.4749	2.1863	1.5652	-5.8013	0.0099
0.8	-0.1307	2.4806	1.1041	-6.5258	-0.0010
0.9	-0.7580	2.8118	0.5852	-7.3560	-0.0132
1	-1.4083	3.1785	0.0000	-8.2901	-0.0246
			B. The case of $\sigma_\pi = 0.6$		
w	p	p_D	p_π	p_θ	p_{mv}
0	6.8209	0.9524	3.8095	-1.0803	0.0000
0.1	8.6479	1.0225	3.6589	-1.1644	-0.0070
0.2	10.9427	1.1069	3.4747	-1.2565	-0.0152
0.3	13.4936	1.2093	3.2539	-1.3589	-0.0256
0.4	15.6019	1.3338	2.9918	-1.4736	-0.0395
0.5	16.2126	1.4825	2.6763	-1.5996	-0.0560
0.6	14.9448	1.6605	2.3011	-1.7382	-0.0764
0.7	12.5443	1.8719	1.8561	-1.8883	-0.1013
0.8	10.0112	2.1212	1.3313	-2.0469	-0.1312
0.9	7.8738	2.4191	0.7181	-2.2140	-0.1696
1	6.2033	2.7752	0.0000	-2.3826	-0.2180

Table 5: Stock Price vs Moving Average Window: $w = 0.1$

The table shows the impact of moving average parameter α on equilibrium stock price, which is $P_t = p + p_D D_t + p_\pi \pi_t + p_\theta \theta_t + p_{mv}(P_t - \alpha A_t)$. The parameters are $r = 0.05, \rho = 0.2, \bar{\pi} = 0.85, \sigma_D = 1.0, \sigma_\pi = 0.8, \sigma_\theta = 3.0, \alpha_\pi = 0.5, \alpha_\theta = 0.4, \alpha_D = 1.0$. The moving average window is measured by $1/\alpha$. The fraction of technical investors is $w = 0.1$.

α	p	p_D	p_π	p_θ	p_{mv}
0.1	20.2743	0.9479	1.6396	-0.3808	0.0234
0.5	21.1403	0.9631	1.6655	-0.3869	0.0083
1	21.6564	0.9668	1.6726	-0.3894	0.0052
5	22.9756	0.9708	1.6822	-0.3936	0.0015
12	23.4179	0.9716	1.6843	-0.3947	0.0007

Table 6: Volatility and risk premium

The table shows the impact of the moving average parameter α on the equilibrium stock price volatility and long run risk premium. The parameters are $r = 0.05, \rho = 0.2, \bar{\pi} = 0.85, \sigma_D = 1.0, \sigma_\pi = 0.8, \sigma_\theta = 3.0, \alpha_\pi = 0.5, \alpha_\theta = 0.4, \alpha_D = 1.0$. The moving average window is measured by $1/\alpha$. The fraction of technical investors is $w = 0.1$.

w	$\alpha = 1$		$\alpha = 0.1$	
	σ_p	RP	σ_p	RP
0	4.0786	0.0282	4.0786	0.0282
0.1	4.1918	0.0158	4.2207	0.0173
0.2	4.3155	0.0036	4.3810	0.0073
0.3	4.4507	-0.0070	4.5620	-0.0008
0.4	4.5987	-0.0140	4.7660	-0.0059
0.5	4.7604	-0.0159	4.9935	-0.0070
0.6	4.9359	-0.0115	5.2434	-0.0036
0.7	5.1244	-0.0003	5.5109	0.0041
0.8	5.3222	0.0176	5.7865	0.0157
0.9	5.5240	0.0423	6.0551	0.0306
1	5.7198	0.0750	6.2951	0.0493

Table 7: Predictive Regression of the S&P500 on MA

The table reports the predictive regression of the S&P500 monthly returns on the moving averages of past daily prices with lag length L days. The data are from January 1963 and December 2012. The Newey-West robust t-statistics are in parentheses and significance at the 1% and 5% levels is given by an ** and an *, respectively.

	α	β	adj. $R^2(\%)$
Moving Average of Daily Returns			
MA(10)	0.006***	2.018**	0.636
	(3.11)	(2.20)	
MA(20)	0.006***	1.255	0.081
	(3.22)	(1.22)	
MA(50)	0.006***	0.892	-0.101
	(3.22)	(0.63)	
MA(100)	0.005***	3.168*	0.294
	(2.78)	(1.66)	
MA(200)	0.006***	1.476	-0.114
	(2.87)	(0.57)	

Table 8: Predictive Regression of the S&P500 on Lagged Returns

The table reports the predictive regression of the S&P500 monthly returns on the lagged monthly returns, and on the moving average of the lagged monthly returns, respectively. The data are from January 1963 and December 2012. The Newey-West robust t-statistics are in parentheses and significance at the 1% and 5% levels is given by an ** and an *, respectively.

	α	β	adj. $R^2(\%)$
Panel A: Past Monthly Returns			
L2	0.006***	-0.026	-0.027
	(3.60)	(-0.85)	
L6	0.006***	-0.047	0.126
	(3.67)	(-1.52)	
L12	0.006***	0.011	-0.085
	(3.46)	(0.35)	
Panel B: Moving Average of Monthly Returns			
MA(2)	0.006***	0.056	0.073
	(3.29)	(1.33)	
MA(6)	0.006***	0.014	-0.094
	(3.36)	(0.19)	
MA(12)	0.005***	0.127	0.054
	(2.88)	(1.25)	

Figure 1: Autocovariance of Price Return

This figure shows the autocovariance of price return defined as

$$< \frac{P_{t+\tau} - P_t}{\tau}, \frac{P_t - P_{t-\tau}}{\tau} >,$$

where τ is the investment horizon. The autocovariance is positive for short horizon $\tau < 0.5$ and become ⟨...⟩ $\sigma_D = 1.0, \sigma_\pi =$ ⟨...⟩ easured by $1/\alpha$.

Autocovariance of Price Return